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ON THE EFFECT OF FLOW DIRECTION ON MIXED CONVECTION FROM A HORIZONTAL CYLINDER

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SUMMARY

The influence of free stream direction on mixed (natural and forced) convective heat transfer from a circular cylinder is investigated. The cylinder, which has an isothermal surface, is placed with its axis horizontal and normal to the oncoming flow. The free stream direction varies between the vertically upward (parallel flow) and the vertically downward (contraflow) directions. The investigation is based on the time integration of the unsteady, two-dimensional equations of motion and energy until reaching steady conditions. The study is limited to Reynolds numbers up to $Re = 40$ and Grashoff numbers of $Gr = Re^2$. The results are compared with the available experimental data and the agreement is satisfactory.

INTRODUCTION

The problem of laminar mixed convective heat transfer from a circular cylinder is a fundamental problem which has received extensive attention because of its many engineering applications. Several experimental studies have been carried out to investigate the effect of different factors on the heat transfer process. Some of these studies resulted in experimental correlations; however, no correlation could successfully predict the overall heat transfer coefficient and take into consideration all the parameters involved in the process.

The first experimental investigation on the influence of free stream direction on the rate of heat transfer from a horizontal cylinder was carried out by Hatton *et al.*[1] who studied the problem up to Reynolds number $Re = 45$ and Grashoff number $Gr = 10$. In their work a correlation based on the vectorial addition of the forced and natural heat transfer coefficients was proposed. The correlation was proved to be successful except for the cases when the forced flow approaches a direction opposite to that of natural convection. Oosthuizen and Madan[2] studied the same problem when the forced flow makes an angle of 0, 90°, 135° or 180° with the direction of natural convection. The study was conducted at relatively high Re and Gr values compared to the range in Reference 1. Other experimental studies are found in References 3–6.

On the other hand, most of the theoretical studies found in the literature have dealt with the case when both forced flow and natural convection are in the same direction. For example, Acrivos[7] studied the problem of combined convection in laminar boundary-layer flow in order to obtain the Nusselt number distribution near a stagnation point for the two cases of $Pr \to 0$ and $Pr \to \infty$. The approach requires the existence of boundary-layer flow and is limited to the region surrounding the stagnation point. Sparrow and Lee[8] obtained an approximate solution for the Nusselt number distribution in the neighbourhood of the forward stagnation point for the case of parallel flow past a circular cylinder. The solution is based on the boundary-layer flow assumption and the problem

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was solved only in the region from the stagnation point up to an angle of 70° where the flow separation was expected to occur. The work by Joshi and Sukhatme[9] and Merkin[10] falls in the same category as Reference 8 since it is again based on the boundary-layer flow assumption and is confined to the region preceding the point of separation. The problems of parallel, cross and contra flow heat transfer from a circular cylinder were also investigated by Nakai and Okazaki.[11] Their work was limited to very small values of *Re* and *Gr* besides its restriction to cases in which either forced or natural convection is dominant. It appears from the literature that there is no complete theoretical solution to the problem of laminar mixed convection from a horizontal cylinder that takes into account the effect of the forced flow direction.

The main objective of this work is to conduct a theoretical investigation on the effect of flow direction on the velocity and thermal fields and consequently on the heat transfer process. Of special interest here is the case when the buoyant forces are of the same order of magnitude as the inertia forces. The study is based on the solution of the full conservation equations of mass, momentum and energy.

PROBLEM STATEMENT AND ASSUMPTIONS

The problem considered here is that of mixed convective heat transfer from a horizontal circular cylinder of radius a. The cylinder surface has a constant temperature T_s and is placed in a uniform stream of velocity u_∞ and temperature T_∞. The direction of the free stream, which is always normal to the cylinder axis, varies from vertically upward ($\gamma = 0$) to vertically downward ($\gamma = 180°$), where γ is the angle between the oncoming flow and the vertically upward direction. The cylinder is assumed to be long enough so that end effects can be neglected and the flow is considered two-dimensional. The fluid properties are also assumed to behave according to the Boussinesq approximations. Consider the line $\theta = 0$ to represent the radius through the rearmost point on the cylinder surface viewed from the upstream direction (see Figure 1). Using the modified polar coordinates (ξ, θ), where $\xi = \ln r$, the governing equations of motion and energy can be written as

$$e^{2\xi}\frac{\partial \zeta}{\partial t} = -\frac{\partial \psi}{\partial \theta}\frac{\partial \zeta}{\partial \xi} + \frac{\partial \psi}{\partial \xi}\frac{\partial \zeta}{\partial \theta} + \frac{2}{Re}\left(\frac{\partial^2 \zeta}{\partial \xi^2} + \frac{\partial^2 \zeta}{\partial \theta^2}\right) - e^{\xi}\frac{Gr}{2Re^2}\left[\frac{\partial \phi}{\partial \xi}\sin(\gamma - \theta) - \frac{\partial \phi}{\partial \theta}\cos(\gamma - \theta)\right] \quad (1)$$

$$e^{2\xi}\zeta = \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \theta^2} \quad (2)$$

$$e^{2\xi}\frac{\partial \phi}{\partial t} = -\frac{\partial \psi}{\partial \theta}\frac{\partial \phi}{\partial \xi} + \frac{\partial \psi}{\partial \xi}\frac{\partial \phi}{\partial \theta} + \frac{2}{Pe}\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \theta^2}\right) \quad (3)$$

where t, r, ψ, ζ, ϕ, Re, Gr and Pe are all dimensionless quantities defined as

$$t = t'u_\infty/a, \quad r = \frac{r'}{a}, \quad \psi = \psi'/au_\infty, \quad \zeta = -\zeta' a/u_\infty$$

$$\phi = (T - T_\infty)/(T_s - T_\infty), \quad Re = 2au_\infty/\nu, \quad Gr = g\beta(2a)^3(T_s - T_\infty)/\nu^2$$

and

$$Pe = Pr\,Re$$

where t' is the time, r' is the radial co-ordinate, ψ' is the stream function, ζ' is the vorticity, T is the temperature, *Re* is the Reynolds number, *Gr* is the Grashoff number, *Pe* is the Peclet number, *Pr* is

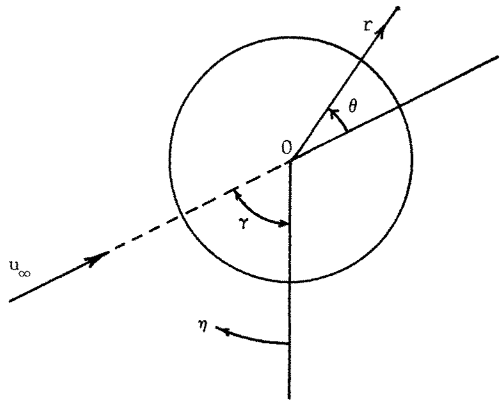

Figure 1. Co-ordinate system

the Prandtl number, β is the coefficient of thermal expansion and ν is the kinematic viscosity. All quantities with primes are dimensional quantities.

The dimensionless velocity components $v_r(=v_r'/u_\infty)$ and $v_\theta(=v_\theta'/u_\infty)$ are related to ψ by

$$v_r = \mathrm{e}^{-\xi}\frac{\partial\psi}{\partial\theta} \quad \text{and} \quad v_\theta = -\mathrm{e}^{-\xi}\frac{\partial\psi}{\partial\xi} \tag{4}$$

The boundary conditions for the velocity and thermal fields are

$$\left.\begin{aligned} &\psi = \frac{\partial\psi}{\partial\theta} = \frac{\partial\psi}{\partial\xi} = 0 \quad \text{and} \quad \phi = 1 \quad \text{at} \quad \xi = 0 \\ &\mathrm{e}^{-\xi}\frac{\partial\psi}{\partial\theta} \to \cos\theta, \quad \mathrm{e}^{-\xi}\frac{\partial\psi}{\partial\xi} \to \sin\theta, \\ &\zeta \to 0 \quad \text{and} \quad \phi \to 0 \quad \text{as} \quad \xi \to \infty \end{aligned}\right\} \tag{5}$$

The conditions in equation (5) are based on the no-slip, impermeability and isothermal conditions on the cylinder surface and the free stream conditions away from it.

METHOD OF SOLUTION

The method used for solving the governing equations (1)–(3) to obtain the steady velocity and temperature distributions is based on studying the time development of the velocity and thermal fields until reaching steady conditions. In this method the uniform stream is assumed to start suddenly from rest at $t=0$ with no temperature difference between the cylinder surface and the oncoming flow. Following this start, the velocity boundary layer develops with time while there is no body force present. At a later time, when the boundary layer thickens, the cylinder surface temperature is assumed to be suddenly raised to T_s. In the subsequent time, the velocity and thermal fields continue to develop simultaneously until the steady conditions are achieved. The approach is similar to that used by Badr.[12]

In the two cases when the forced flow is either vertically upward or vertically downward ($\gamma = 0$ or 180°) the streamlines and isotherms are symmetrical about a vertical plane passing by the cylinder axis. However, for any other value of γ the velocity and temperature fields become asymmetric. In general, we express the functions ψ, ζ and ϕ in terms of the Fourier series expansions as

$$\left.\begin{aligned} \psi &= \tfrac{1}{2}F_0(\xi,t) + \sum_{n=1}^{N} f_n(\xi,t)\sin n\theta + F_n(\xi,t)\cos n\theta \\ \zeta &= \tfrac{1}{2}G_0(\xi,t) + \sum_{n=1}^{N} g_n(\xi,t)\sin n\theta + G_n(\xi,t)\cos n\theta \\ \phi &= \tfrac{1}{2}H_0(\xi,t) + \sum_{n=1}^{N} h_n(\xi,t)\sin n\theta + H_n(\xi,t)\cos n\theta \end{aligned}\right\} \tag{6}$$

For the two special cases of $\gamma = 0$ and $\gamma = 180°$ the functions F_0, F_n, G_0, G_n and h_n will vanish. The number of terms N in the series depends on the values of Re and Gr. The solution starts, in all cases, with $N = 2$ and then one more term is added when the last non-zero term in any of the series reaches a certain small value. However the maximum number of terms used in all cases is 20. Substituting from equation (6) into equation (1) and integrating both sides of the resulting equation (after multiplying each side, once at a time by 1, $\sin n\theta$, $\cos n\theta$) with respect to θ between the limits of 0 and 2π, we obtain

$$e^{2\xi}\frac{\partial G_0}{\partial t} - \frac{2}{Re}\frac{\partial^2 G_0}{\partial \xi^2} = X_0(\gamma,\xi,t) \tag{7a}$$

$$2e^{2\xi}\frac{\partial g_n}{\partial t} - \frac{4}{Re}\left(\frac{\partial^2 g_n}{\partial \xi^2} - n^2 g_n\right) - nF_n\frac{\partial G_0}{\partial \xi} + nG_n\frac{\partial F_0}{\partial \xi} = X_{n_1}(\gamma,\xi,t) \tag{7b}$$

$$2e^{2\xi}\frac{\partial G_n}{\partial t} - \frac{4}{Re}\left(\frac{\partial^2 G_n}{\partial \xi^2} - n^2 G_n\right) + nf_n\frac{\partial G_0}{\partial \xi} - ng_n\frac{\partial F_0}{\partial \xi} = X_{n_2}(\gamma,\xi,t) \tag{7c}$$

where $F_n = F_n(\xi,t)$, $G_n = G_n(\xi,t), \ldots$, etc., and the functions X_0, X_{n_1} and X_{n_2} are defined as

$$\begin{aligned} X_0(\gamma,\xi,t) = {} & e^{\xi}\frac{Gr}{2Re^2}\left[\sin\gamma\left(\frac{\partial H_1}{\partial \xi} + H_1\right) - \cos\gamma\left(\frac{\partial h_1}{\partial \xi} + h_1\right)\right] \\ & + \sum_{n=1}^{N} n\left(F_n\frac{\partial g_n}{\partial \xi} - f_n\frac{\partial G_n}{\partial \xi} + g_n\frac{\partial F_n}{\partial \xi} - G_n\frac{\partial f_n}{\partial \xi}\right) \end{aligned} \tag{8a}$$

$$\begin{aligned} X_{n_1}(\gamma,\xi,t) = {} & e^{\xi}\frac{Gr}{2Re^2}\left[\sin\gamma\left\{\frac{\partial h_{n-1}}{\partial \xi} + \frac{\partial h_{n+1}}{\partial \xi} - (n-1)h_{n-1} + (n+1)h_{n+1}\right\}\right. \\ & \left. + \cos\gamma\left\{-\delta_n\frac{\partial H_0}{\partial \xi} + \frac{\partial H_{n+1}}{\partial \xi} - \frac{\partial H_{n-1}}{\partial \xi} + (n-1)H_{n-1} + (n+1)H_{n+1}\right\}\right] \\ & + \sum_{m=1}^{N}\left\{\frac{\partial g_m}{\partial \xi}(Kf_K - Jf_J) + \frac{\partial G_m}{\partial \xi}[KF_K - (m-n)F_J]\right. \\ & \left. + mg_m\left[\frac{\partial f_K}{\partial \xi} - \operatorname{sgn}(m-n)\frac{\partial f_J}{\partial \xi}\right] + mG_m\left(\frac{\partial F_K}{\partial \xi} - \frac{\partial F_J}{\partial \xi}\right)\right\} \end{aligned} \tag{8b}$$

$$\begin{aligned} X_{n_2}(\gamma,\xi,t) = {} & e^{\xi}\frac{Gr}{2Re^2}\left[\sin\gamma\left\{\delta_n\frac{\partial H_0}{\partial \xi} + \frac{\partial H_{n+1}}{\partial \xi} + \frac{\partial H_{n-1}}{\partial \xi} + (n+1)H_{n+1} - (n-1)H_{n-1}\right\}\right. \\ & \left. + \cos\gamma\left\{\frac{\partial h_{n-1}}{\partial \xi} - \frac{\partial h_{n+1}}{\partial \xi} - (n-1)h_{n-1} - (n+1)h_{n+1}\right\}\right] \end{aligned}$$

$$+\sum_{m=1}^{N}\left\{\frac{\partial g_m}{\partial \xi}[KF_K+(m-n)F_J]-\frac{\partial G_m}{\partial \xi}(Kf_K+Jf_J)\right.$$
$$\left.+mg_m\left[\frac{\partial F_K}{\partial \xi}+\frac{\partial F_J}{\partial \xi}\right]-mG_m\left[\frac{\partial f_K}{\partial \xi}+\text{sgn}(m-n)\frac{\partial f_J}{\partial \xi}\right]\right\} \tag{8c}$$

where $K=m+n, j=|m-n|$, $\delta_n=1$ when $n=1$ and $\delta_n=0$ when $n\neq 1$, and $\text{sgn}(m-n)$ means the sign of the term $(m-n)$.

If we again substitute from equation (6) into equation (2) and use simple mathematical analysis we obtain

$$\frac{\partial^2 F_0}{\partial \xi^2}=e^{2\xi}G_0 \tag{9a}$$

$$\frac{\partial^2 f_n}{\partial \xi^2}-n^2 f_n=e^{2\xi}g_n \tag{9b}$$

$$\frac{\partial^2 F_n}{\partial \xi^2}-n^2 F_n=e^{2\xi}G_n \tag{9c}$$

Finally by using equation (6) together with equation (3) we obtain the following set of partial differential equations (p.d.e.s.) for the functions H_0, h_n and H_n

$$e^{2\xi}\frac{\partial H_0}{\partial t}-\frac{2}{Pe}\frac{\partial^2 H_0}{\partial \xi^2}=Z_0(\xi,t) \tag{10a}$$

$$2e^{2\xi}\frac{\partial h_n}{\partial t}-\frac{4}{Pe}\left(\frac{\partial^2 h_n}{\partial \xi^2}-n^2 h_n\right)-nF_n\frac{\partial H_0}{\partial \xi}+nH_n\frac{\partial F_0}{\partial \xi}=Z_{n_1}(\xi,t) \tag{10b}$$

$$2e^{2\xi}\frac{\partial H_n}{\partial t}-\frac{4}{Pe}\left(\frac{\partial^2 H_n}{\partial \xi^2}-n^2 H_n\right)+nf_n\frac{\partial H_0}{\partial \xi}-nh_n\frac{\partial F_0}{\partial \xi}=Z_{n_2}(\xi,t) \tag{10c}$$

where the functions Z_0, Z_{n_1} and Z_{n_2} can be easily defined.

The boundary conditions for all the variables given in equations (7), (9) and (10) can be obtained by using equations (5) and (6) which results in

$$\left.\begin{aligned}&F_0=f_n=F_n=h_n=H_n=0\\&\frac{\partial F_0}{\partial \xi}=\frac{\partial f_n}{\partial \xi}=\frac{\partial F_n}{\partial \xi}=0\quad\text{and}\quad H_0=2\end{aligned}\right\}\quad\text{at}\quad \xi=0 \tag{11a}$$

and

$$\left.\begin{aligned}&F_0, F_n, G_0, g_n, G_n, H_0, h_n \text{ and } H_n\to 0\\&f_n\to\delta_n e^{\xi}\end{aligned}\right\}\quad\text{as}\quad \xi\to\infty \tag{11b}$$

By integrating both sides of equations (9a), (9b) and (9c) with respect to ξ from $\xi=0$ to $\xi=\infty$ and applying the conditions given in (11), the following integral conditions can be deduced

$$\int_0^\infty e^{2\xi}G_0\,d\xi=0 \tag{12a}$$

$$\int_0^\infty e^{(2-n)\xi}g_n\,d\xi=2\delta_n \tag{12b}$$

$$\int_0^\infty e^{(2-n)\xi}G_n\,d\xi=0 \tag{12c}$$

The above conditions were used to determine the values of the functions G_0, g_n and G_n at $\xi = 0$. It is also important to mention that satisfying the integral condition (12a) is necessary to ensure the continuity of pressure around the cylinder surface.

In the first part of the motion (following $t = 0$) the streamlines are symmetric about the line $\theta = 0$ since the buoyancy force is zero. In this part the boundary-layer co-ordinate x, where $x = \xi/2(2t/Re)^{1/2}$, is used instead of ξ since the boundary-layer thickness is very small. The method of solution in this part is exactly the same as that used by Badr and Dennis[13] for the special case of no cylinder rotation. The integration process in this part is terminated at a time $t = Re/8$, at which $\xi = x$, and the boundary layer becomes thick enough to use the original co-ordinate ξ. At the start of the second part of the motion ($t = Re/8$) the cylinder surface temperature is suddenly increased to T_s, allowing both velocity and thermal boundary layers to develop with time. In this part the three sets of p.d.e.s [equations (7), (9) and (10)] are integrated to advance the solution of ψ, ζ and ϕ in time.

The numerical method used for integrating equations (7) and (10) is based on a Crank–Nicolson finite-difference scheme in order to obtain the functions G_0, g_n, G_n, H_0, h_n and H_n at time $t + \Delta t$ provided that all of these functions are known at time t. At each time step a direct solution for equation (9a) is obtained using central differences to determine the function F_0. The functions f_n and F_n are obtained by solving equations (9b) and (9c) using a step-by-step integration scheme similar to that used by Dennis and Chang.[14] The solution procedure is similar to that used by Badr[12] for tackling the problem of cross mixed convection.

RESULTS AND DISCUSSION

The effect of flow direction on the rate of heat transfer from a horizontal cylinder is studied for Reynolds numbers of $Re = 5, 10, 20$ and 40 and Grashoff numbers of $Gr = Re^2$. The flow direction is varied from $\gamma = 0$ to $\gamma = 180°$ with steps of 30°. To compare and discuss results let us define the local and average coefficients of heat transfer h and $\bar{h}$ as

$$h = \dot{q}/(T_s - T_\infty) \quad \text{and} \quad \bar{h} = \frac{1}{2\pi}\int_0^{2\pi} h\,\mathrm{d}\theta \tag{13}$$

where $\dot{q} = -k(\partial T/\partial r')_{r'=a}$ is the rate of heat transfer per unit area. Define also the local and average Nusselt numbers Nu and $\overline{Nu}$ such that

$$Nu = 2ah/k \quad \text{and} \quad \overline{Nu} = 2a\bar{h}/k \tag{14}$$

where k is the thermal conductivity. The relationship between each of Nu and $\overline{Nu}$ and the functions H_0, h_n and H_n can be deduced from equations (6), (13) and (14) and written as

$$Nu = -2\left[\frac{1}{2}\frac{\partial H_0}{\partial \xi} + \sum_{n=1}^{N}\left\{\frac{\partial h_n}{\partial \xi}\sin n\theta + \frac{\partial H_n}{\partial \xi}\cos n\theta\right\}\right]_{\xi=0} \tag{15a}$$

$$\overline{Nu} = -\left(\frac{\partial H_0}{\partial \xi}\right)_{\xi=0} \tag{15b}$$

It is found from the results that the velocity field is highly influenced by the free stream direction. Figure 2 shows the vorticity distribution on the cylinder surface for the case of $Re = 20$, $Gr = 400$ and $Pr = 0{\cdot}7$ and at different flow directions ($\gamma = 0$, 60°, 120° and 180°). It can be seen from the Figure that $|\zeta|$ decreases over most of the cylinder surface as γ increases. This is mainly due to the fact that when $\gamma = 0$ the buoyancy forces are aiding the flow and accordingly causing higher velocities near the cylinder surface. This effect decreases as γ increases until reaching $\gamma = 180°$ at

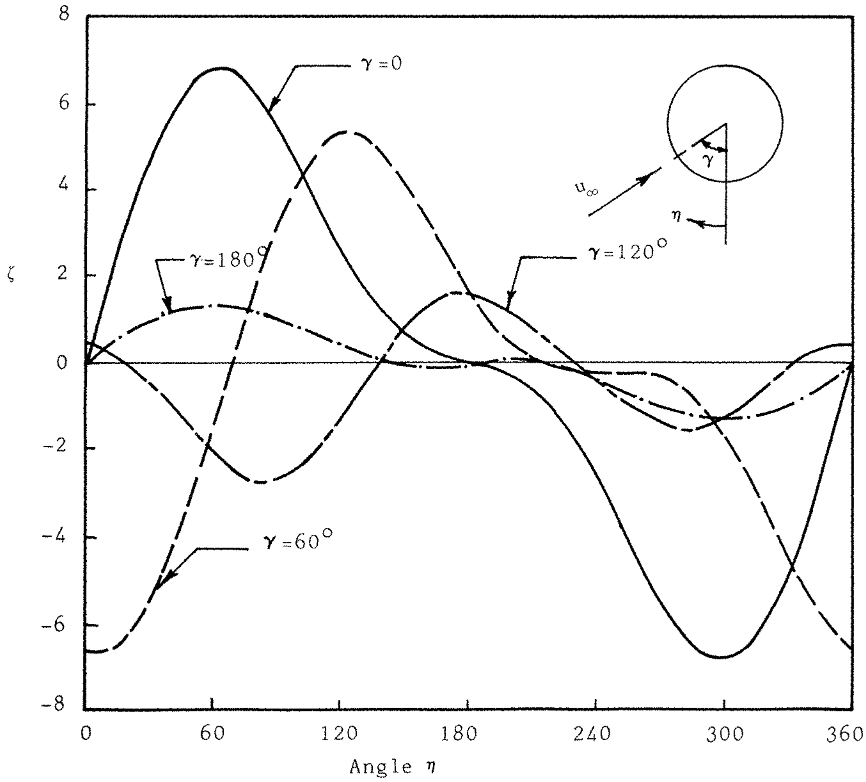

Figure 2. The variation of the vorticity on the cylinder surface for different free stream directions ($Re = 20$, $Gr = 400$ and $Pr = 0{\cdot}7$)

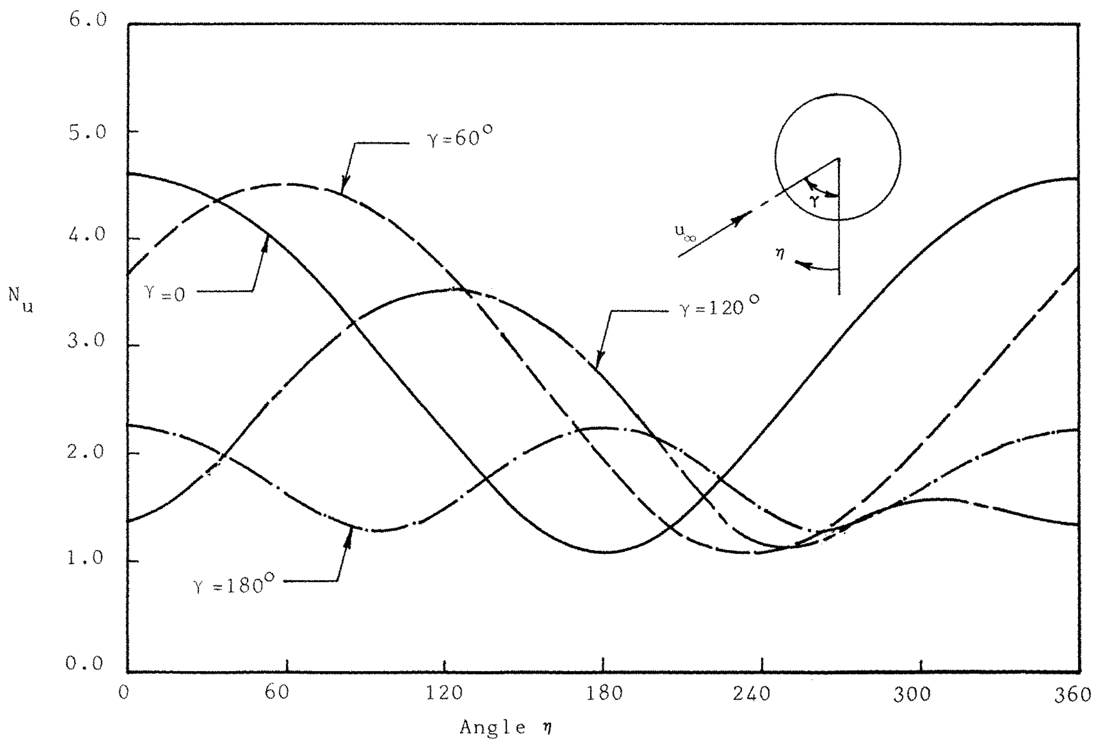

Figure 3. The Nusselt number distribution on the cylinder surface for different free stream directions ($Re = 20$, $Gr = 400$ and $Pr = 0{\cdot}7$)

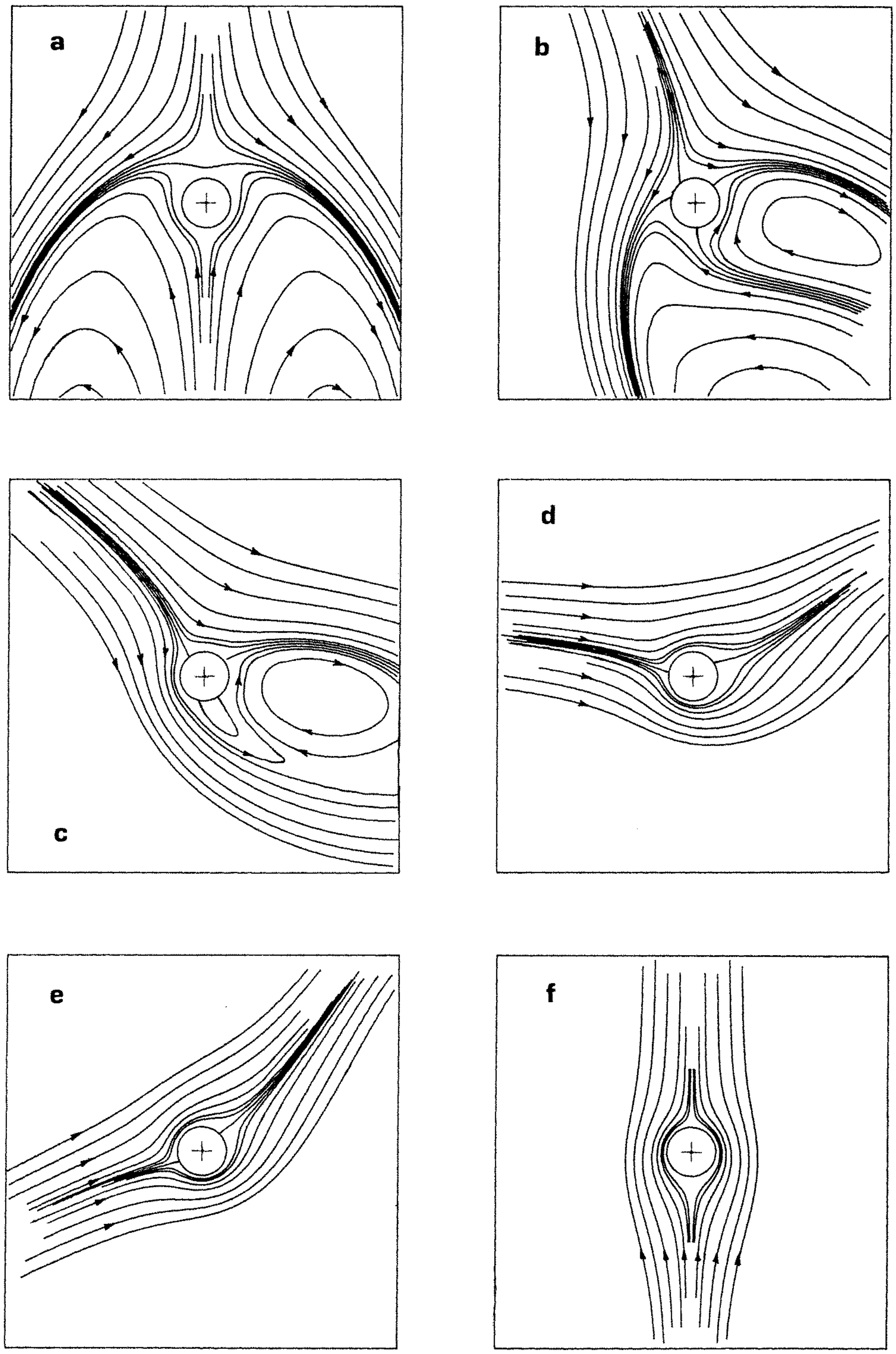

Figure 4. Streamline patterns for $Re = 10$, $Pr = 0{\cdot}7$, $Gr = 100$ and for values of γ of (a) 180°, (b) 150°, (c) 120°, (d) 90°, (e) 60° and (f) 0° (streamlines plotted are $\psi = -2{\cdot}0, -1{\cdot}5, -1{\cdot}0, -0{\cdot}5, -0{\cdot}25, -0{\cdot}1, -0{\cdot}05, 0, 0{\cdot}05, 0{\cdot}1, 0{\cdot}25, 0{\cdot}5, 1{\cdot}0, 1{\cdot}5$ and $2{\cdot}0$)

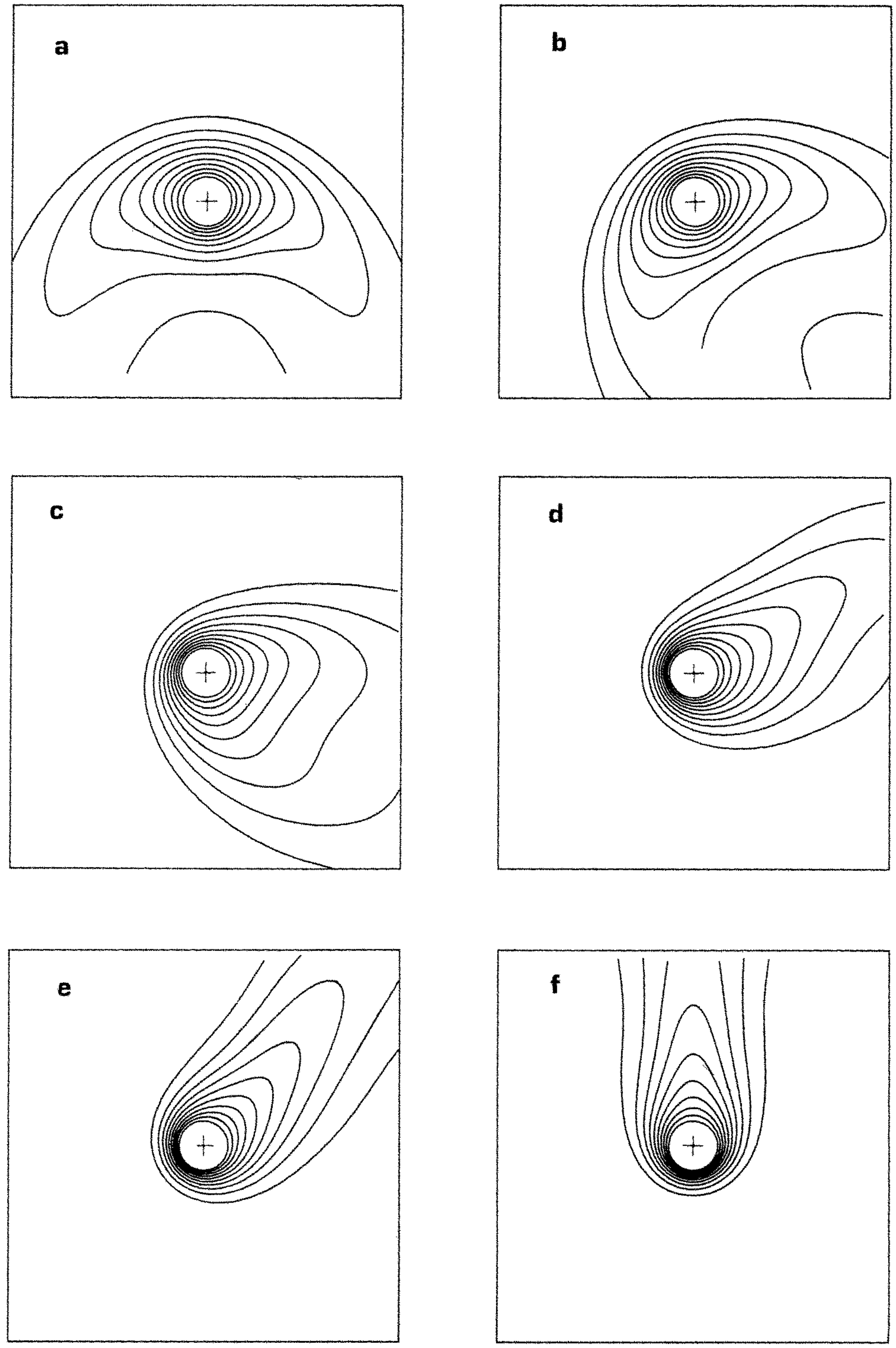

Figure 5. Isotherm patterns for $Re = 10$, $Pr = 0{\cdot}7$, $Gr = 100$ and for values of γ of (a) 180°, (b) 150°, (c) 120°, (d) 90°, (e) 60° and (f) 0° (isotherms plotted are $\phi = 0{\cdot}1, 0{\cdot}2, \ldots, 1{\cdot}0$)

which the direction of flow induced by natural convection is directly opposite to the forced flow direction.

It is also found that the local Nusselt number distribution is greatly affected by the direction of the main stream. The variation of Nu around the cylinder surface for the same case and at different values of γ can be seen in Figure 3. The Figure shows that Nu decreases with the increase of γ over

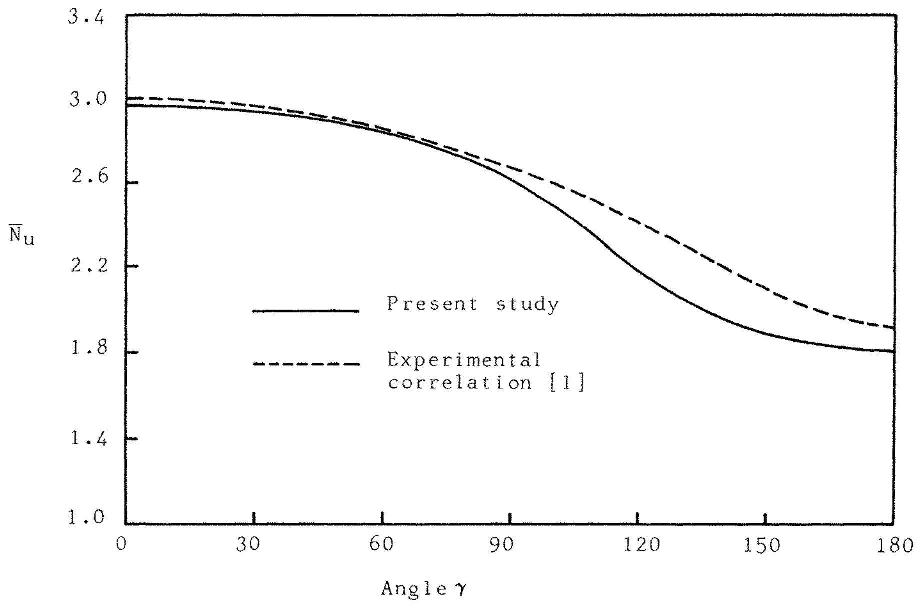

Figure 6. Comparison between the values of $\overline{Nu}$ obtained from the present work and Hatton's experimental correlation[1] for the case of $Re = 20$, $Pr = 0{\cdot}7$ and $Gr = 400$

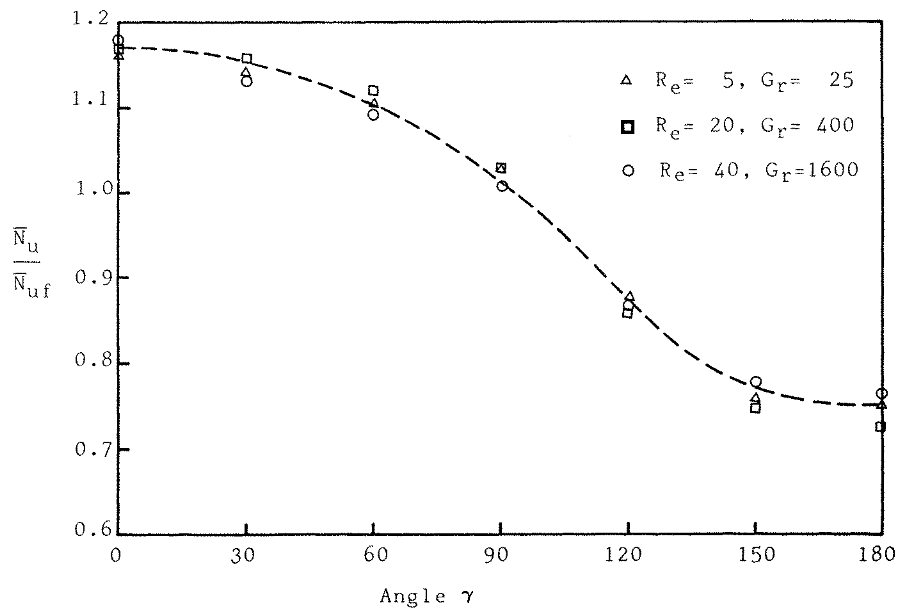

Figure 7. Variation of the ratio $\overline{Nu}/\overline{Nu}_f$ with free stream direction

most of the cylinder surface. This is expected to occur as a natural result of the slow down of the flow velocity near the surface. Figure 4 and 5 show the streamline and isotherm patterns for the case of $Re = 10$ and different values of γ. It can be seen from Figure 4 that the size and orientation of the wake region is completely dependent on γ. When $\gamma = 0$ the circulating flow zone in the wake disappears and flow separation occurs only at the backward stagnation point ($\eta = 180°$). On the other hand, when $\gamma = 180°$ the flow field is divided into two distinct zones. The first one is surrounding the cylinder with the enclosed circulating flow driven by the buoyancy forces. The heat transfer regime in this zone is dominated by natural convection. The second zone is outside the first one and the buoyant forces there have smaller effect. The heat transfer between the two zones is mainly due to conduction at the boundaries since the fluid in the first zone is never convected to the second one.

A comparison between the present results for $\overline{Nu}$ and the experimental correlation obtained by Hatton *et al.*[1] can be seen in Figure 6. The agreement between the two is satisfactory up to $\gamma = 90°$, however as γ approaches 180° Hatton's correlation becomes inapplicable.[1] Figure 7 shows the variation of $\overline{Nu}/\overline{Nu_f}$ with the angle γ for all the cases considered, where $\overline{Nu_f}$ is the average Nusselt number for a forced convection regime. The Figure also shows that the maximum value of $\overline{Nu}$ occurs when $\gamma = 0$ and the minimum value occurs when $\gamma = 180°$ provided that the flow direction is the only variable.

CONCLUSION

The effect of flow direction on mixed convection heat transfer from a horizontal circular cylinder has been studied in the range of Reynolds numbers up to $Re = 40$ and for Grashoff numbers of $Gr = Re^2$. A numerical method was used to integrate the unsteady two-dimensional equations of motion and energy. The variation of the average Nusselt number with flow direction was found to be in good agreement with the previous experimental results especially when $\gamma < 90°$. The method used was found to be highly stable and accurate in the considered range of *Re* and *Gr*. The streamlines and isotherms were plotted to show the effect of flow direction on the velocity and thermal fields.

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